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2	.....	,
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1	.....	
2	.....	
•	.....	10
,	.....	11
,	.....	12
,	.....	1
1	.....	1
2	.....	22
•	.....	2
,	.....	2
,	.....	•1
,	.....	•
,	.....	,0
,	.....	,•
1	.....	,•
2	.....	,•
•	.....(.)	1
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,	.....	0
,	.....	1
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1	.....	•
2	.....	,
•	.....	•
10	.....	2
1	.....	2
2	.....	•
•	.....	•
11	.....	0
1	.....	0
2	.....	1
12	.....	2
1	.....	2
2	.....	•
1•	.....	•
1,	.....	•
1	.....	•



Example 9.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 10.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 11.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 12.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 13.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 14.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 15.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 16.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 17.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 18.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 19.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

Example 20.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$

$$\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$$

Example 21.  $\int_{-1}^1 x^2 dx = \frac{1}{3}x^3 \Big|_{-1}^1 = \frac{1}{3}(1^3 - (-1)^3) = \frac{2}{3}$



The first step is to determine the total number of units produced. This is done by adding the units in inventory at the beginning of the period to the units produced during the period, and then subtracting the units in inventory at the end of the period.

Step 1: Determine the total number of units produced.

Step 1:  $2,200,000$  units produced.

Step 20:  $1,100,000$  units produced.

The second step is to determine the cost per unit. This is done by dividing the total cost of production by the total number of units produced.

Step 1:  $2,200,000$  units produced.

Step 1:  $1,200,000$  units produced.

Frage 21

Die Funktion  $f: \mathbb{R} \rightarrow \mathbb{R}$  ist durch  $f(x) = \frac{1}{2}x^2 + 3x - 5$  gegeben. Bestimmen Sie die Nullstellen von  $f$ .

Die Nullstellen einer quadratischen Gleichung  $ax^2 + bx + c = 0$  sind gegeben durch die Mitternachtsformel  $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . In diesem Fall ist  $a = \frac{1}{2}$ ,  $b = 3$  und  $c = -5$ . Die Diskriminante ist  $\Delta = 3^2 - 4 \cdot \frac{1}{2} \cdot (-5) = 9 + 10 = 19$ . Die Nullstellen sind  $x_1 = \frac{-3 + \sqrt{19}}{1}$  und  $x_2 = \frac{-3 - \sqrt{19}}{1}$ .

Frage 22

Die Funktion  $f: \mathbb{R} \rightarrow \mathbb{R}$  ist durch  $f(x) = x^3 - 3x^2 + 2x$  gegeben. Bestimmen Sie die Nullstellen von  $f$ .

Die Nullstellen einer kubischen Gleichung  $ax^3 + bx^2 + cx + d = 0$  können durch Faktorisierung gefunden werden. In diesem Fall ist  $f(x) = x(x^2 - 3x + 2) = x(x-1)(x-2)$ . Die Nullstellen sind  $x_1 = 0$ ,  $x_2 = 1$  und  $x_3 = 2$ .

Frage 23

Die Funktion  $f: \mathbb{R} \rightarrow \mathbb{R}$  ist durch  $f(x) = x^2 - 4x + 4$  gegeben. Bestimmen Sie die Nullstellen von  $f$ .

$$\Delta = b^2 - 4ac = 4^2 - 4 \cdot 1 \cdot 4 = 16 - 16 = 0$$

Frage 24

Die Funktion  $f: \mathbb{R} \rightarrow \mathbb{R}$  ist durch  $f(x) = x^2 - 5x + 6$  gegeben. Bestimmen Sie die Nullstellen von  $f$ .

- (1)  $x_1 = 2, x_2 = 3$
- (2)  $x_1 = 3, x_2 = 2$
- (3)  $x_1 = 2, x_2 = 4$
- (4)  $x_1 = 3, x_2 = 4$
- (5)  $x_1 = 2, x_2 = 5$

Frage 25

Die Funktion  $f: \mathbb{R} \rightarrow \mathbb{R}$  ist durch  $f(x) = x^2 - 7x + 12$  gegeben. Bestimmen Sie die Nullstellen von  $f$ .

Frage 26

Die Funktion  $f: \mathbb{R} \rightarrow \mathbb{R}$  ist durch  $f(x) = x^2 - 8x + 15$  gegeben. Bestimmen Sie die Nullstellen von  $f$ .

1. The first part of the paper is devoted to the study of the asymptotic behavior of the solutions of the system of differential equations

as  $t \rightarrow \infty$ . In the second part we study the asymptotic behavior of the solutions of the system of differential equations

... (1) ... (2) ...

... (1) ... (2) ...

... (1) ... (2) ...

•0

... (1) ... (2) ...

... (1) ... (2) ...

... (1) ... (2) ...

•1

... (1) ... (2) ...

(1) ... (2) ...

(2) ... (1) ... (2) ...

1. ... (1) ... (2) ...



•  $\frac{1}{2} \times 100 = 50\%$

•  $0$

•  $100$

$$4 \times 10^4 = 40000$$

•  $100$

•  $100$

•  $100$

•  $100$

- (1)  $100$
- (2)  $100$
- (3)  $100$

(1)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (normalization condition)

Let's consider the function  $\delta(x)$  as a limit of a sequence of functions  $\delta_n(x)$  which are zero outside the interval  $[-n, n]$  and have a peak of height  $1/n$  at  $x=0$ . (For example,  $\delta_n(x) = \frac{1}{2n} \exp(-|x|/n)$  for  $|x| \leq n$  and 0 otherwise).

Example:  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (normalization condition)

(1)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (normalization condition)

(2)  $\int_{-\infty}^{\infty} \delta(x) x dx = 0$  (center of mass)

(3)  $\int_{-\infty}^{\infty} \delta(x) x^2 dx = 0$  (second moment)

(4)  $\int_{-\infty}^{\infty} \delta(x) x^n dx = 0$  for  $n > 0$  (higher moments)

(5)  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$  (sifting property)

(6)  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$  (sifting property)

$$\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$$

Example:  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$

Let's consider the function  $\delta(x)$  as a limit of a sequence of functions  $\delta_n(x)$  which are zero outside the interval  $[-n, n]$  and have a peak of height  $1/n$  at  $x=0$ .

Example:  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$

exmp, 1

... ..

- (1) ... ..
- (2) ... ..
- (\*) ... ..
- (3) ... ..
- (4) ... ..
- (5) ... ..
- (6) ... ..
- (7) ... ..

... ..

exmp, 2

... ..

... ..

exmp, 3

... ..

... ..

- (1) ... .. (2) ... .. (\*) ... ..
- (2) ... ..
- (\*) ... ..

Example 1

Two identical particles of mass  $m$  are suspended from a common point by two strings of length  $l$ . The particles are released from rest at an angle  $\theta$  to the vertical. Find the speed of each particle just before they collide.

Two identical particles of mass  $m$  are suspended from a common point by two strings of length  $l$ . The particles are released from rest at an angle  $\theta$  to the vertical. Find the speed of each particle just before they collide.

- (1) ...  $\$2.0$  ...
- (2) ...
- (3) ...
- (4) ...
- (5) ...
- (6) ...

Two identical particles of mass  $m$  are suspended from a common point by two strings of length  $l$ . The particles are released from rest at an angle  $\theta$  to the vertical. Find the speed of each particle just before they collide.

Example 2

Two identical particles of mass  $m$  are suspended from a common point by two strings of length  $l$ . The particles are released from rest at an angle  $\theta$  to the vertical. Find the speed of each particle just before they collide.

Example 3

Two identical particles of mass  $m$  are suspended from a common point by two strings of length  $l$ . The particles are released from rest at an angle  $\theta$  to the vertical. Find the speed of each particle just before they collide.

Example 1:  $f(x) = x^2 + 2x + 1$ . The function is a parabola opening upwards with its vertex at  $(-1, 0)$ . The x-axis is a line of symmetry. The function is zero at  $x = -1$ .

Example 2:  $f(x) = x^2 - 4x + 4$ . The function is a parabola opening upwards with its vertex at  $(2, 0)$ . The x-axis is a line of symmetry. The function is zero at  $x = 2$ .

Example 3:  $f(x) = x^2 + 1$ . The function is a parabola opening upwards with its vertex at  $(0, 1)$ . The y-axis is a line of symmetry. The function is never zero.

Example 4:  $f(x) = x^2 - 1$ . The function is a parabola opening upwards with its vertex at  $(0, -1)$ . The y-axis is a line of symmetry. The function is zero at  $x = 1$  and  $x = -1$ .

Example 5:  $f(x) = x^2 + 2x - 3$ . The function is a parabola opening upwards with its vertex at  $(-1, -4)$ . The x-axis is a line of symmetry. The function is zero at  $x = 1$  and  $x = -3$ .

Example 6:  $f(x) = x^2 - 2x - 3$ . The function is a parabola opening upwards with its vertex at  $(1, -4)$ . The x-axis is a line of symmetry. The function is zero at  $x = 3$  and  $x = -1$ .

(1)  $f(x) = x^2 + 2x + 1 = (x+1)^2$ . The function is a parabola opening upwards with its vertex at  $(-1, 0)$ . The x-axis is a line of symmetry. The function is zero at  $x = -1$ .

(2)  $f(x) = x^2 - 4x + 4 = (x-2)^2$ . The function is a parabola opening upwards with its vertex at  $(2, 0)$ . The x-axis is a line of symmetry. The function is zero at  $x = 2$ .

(3)  $f(x) = x^2 + 1$ . The function is a parabola opening upwards with its vertex at  $(0, 1)$ . The y-axis is a line of symmetry. The function is never zero.

(1)  $\int_0^1 x^2 dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3} (1^3 - 0^3) = \frac{1}{3}$

$\int_0^1 x^3 dx = \frac{1}{4} x^4 \Big|_0^1 = \frac{1}{4} (1^4 - 0^4) = \frac{1}{4}$

(2)  $\int_0^1 x^2 dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3} (1^3 - 0^3) = \frac{1}{3}$

(3)  $\int_0^1 x^3 dx = \frac{1}{4} x^4 \Big|_0^1 = \frac{1}{4} (1^4 - 0^4) = \frac{1}{4}$

(4)  $\int_0^1 x^4 dx = \frac{1}{5} x^5 \Big|_0^1 = \frac{1}{5} (1^5 - 0^5) = \frac{1}{5}$

Example 1:  $\int_0^1 x^2 dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3} (1^3 - 0^3) = \frac{1}{3}$

Example 2:  $\int_0^1 x^3 dx = \frac{1}{4} x^4 \Big|_0^1 = \frac{1}{4} (1^4 - 0^4) = \frac{1}{4}$

$\int_0^1 x^4 dx = \frac{1}{5} x^5 \Big|_0^1 = \frac{1}{5} (1^5 - 0^5) = \frac{1}{5}$



2.  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx$  (using the residue theorem)

(1)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \int_{-\infty}^{\infty} \frac{1}{(x-i)(x+i)} dx$

(2)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \int_{-\infty}^{\infty} \frac{1}{(x-i)(x+i)} dx$   
 Consider the contour  $C$  in the complex plane consisting of the real axis from  $-R$  to  $R$  and the upper semicircle  $\Gamma_R$  of radius  $R$  in the upper half-plane.

(3)  $\int_C \frac{1}{z^2+1} dz = \int_{-R}^R \frac{1}{x^2+1} dx + \int_{\Gamma_R} \frac{1}{z^2+1} dz$

(4)  $\int_C \frac{1}{z^2+1} dz = 2\pi i \operatorname{Res}_{z=i} \frac{1}{z^2+1}$

(5)  $\int_C \frac{1}{z^2+1} dz = \pi$

(6)  $\int_{-R}^R \frac{1}{x^2+1} dx + \int_{\Gamma_R} \frac{1}{z^2+1} dz = \pi$

(7)  $\int_{-R}^R \frac{1}{x^2+1} dx = \pi - \int_{\Gamma_R} \frac{1}{z^2+1} dz$

(8)  $\lim_{R \rightarrow \infty} \int_{-R}^R \frac{1}{x^2+1} dx = \pi$

(9)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$

(10)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$

(11)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
 (12)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
 (13)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
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 (20)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$

(21)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
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 (24)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
 (25)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
 (26)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
 (27)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
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(31)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
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 (50)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$

(51)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
 (52)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$   
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 (60)  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \pi$

- (1)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  (if  $f$  is continuous at  $a$ )
- (2)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  (if  $f$  is continuous at  $a$ )
- (3)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  (if  $f$  is continuous at  $a$ )
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Example:  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  (if  $f$  is continuous at  $a$ )



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- (1) ...
- (2) ...
- (\*) ...

Frage 2

Die Nachfragefunktion für ein Produkt lautet  $N(x) = 100 - 2x$ . Die Kostenfunktion lautet  $K(x) = 20x + 0,01x^2$ . Die Erlösfunktion lautet  $E(x) = 100x - 2x^2$ . Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist. Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.

- (1) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
- (2) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
- (3) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
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- (6) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
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- (8) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
- (9) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.

Frage 2 - Die Nachfragefunktion für ein Produkt lautet  $N(x) = 100 - 2x$ . Die Kostenfunktion lautet  $K(x) = 20x + 0,01x^2$ . Die Erlösfunktion lautet  $E(x) = 100x - 2x^2$ .

Frage 3

Die Nachfragefunktion für ein Produkt lautet  $N(x) = 100 - 2x$ . Die Kostenfunktion lautet  $K(x) = 20x + 0,01x^2$ . Die Erlösfunktion lautet  $E(x) = 100x - 2x^2$ . Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.

- (1) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
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- (8) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
- (9) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
- (10) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
- (11) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.
- (12) Die Erlösfunktion ist eine Parabel, die nach unten geöffnet ist.

(1.)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (normalization condition)  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$

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(.)  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$  (property of delta function)  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$

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Example:  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$  (property of delta function)  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$

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Example:  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$  (property of delta function)  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$





Example 1: The following are the results of the first two tests of the hypothesis that the population mean is equal to 100. The first test is a one-sample t-test and the second is a two-sample t-test.

- (1)  $t = 2.5$ ,  $p = 0.01$ ,  $n = 10$ ,  $s = 10$ ,  $\mu = 100$ ,  $\sigma = 10$
- (2)  $t = 1.5$ ,  $p = 0.05$ ,  $n = 10$ ,  $s = 10$ ,  $\mu = 100$ ,  $\sigma = 10$
- (3)  $t = 2.5$ ,  $p = 0.01$ ,  $n = 10$ ,  $s = 10$ ,  $\mu = 100$ ,  $\sigma = 10$
- (4)  $t = 1.5$ ,  $p = 0.05$ ,  $n = 10$ ,  $s = 10$ ,  $\mu = 100$ ,  $\sigma = 10$

3. The following are the results of the first two tests of the hypothesis that the population mean is equal to 100. The first test is a one-sample t-test and the second is a two-sample t-test.

Example 2: The following are the results of the first two tests of the hypothesis that the population mean is equal to 100. The first test is a one-sample t-test and the second is a two-sample t-test.

- (1)  $t = 2.5$ ,  $p = 0.01$ ,  $n = 10$ ,  $s = 10$ ,  $\mu = 100$ ,  $\sigma = 10$

The following are the results of the first two tests of the hypothesis that the population mean is equal to 100. The first test is a one-sample t-test and the second is a two-sample t-test.

The following are the results of the first two tests of the hypothesis that the population mean is equal to 100. The first test is a one-sample t-test and the second is a two-sample t-test.

(2)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^-) + f(a^+))$ .

(3)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^-) + f(a^+))$ .

(4)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^-) + f(a^+))$ .  
If  $f$  has a removable discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$ .  
If  $f$  has an essential discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx$  is not defined.

(5)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^-) + f(a^+))$ .

$\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^-) + f(a^+))$ .

$\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^-) + f(a^+))$ .  
If  $f$  has a removable discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$ .  
If  $f$  has an essential discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx$  is not defined.

$\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^-) + f(a^+))$ .

exp 0 ... (mathematical derivation) ...

exp 1 ... (mathematical derivation) ...

exp 2 ... (mathematical derivation) ...

1)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (normalization)  
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$  (sifting property)  
 $\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$  (sifting property)

Example

2)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$   
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$   
 $\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$   
 20

3)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$   
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$

4)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$   
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$

Example

1)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$

(1)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$

(2)  $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$

(3)  $\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$

(4)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$   
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$   
 $\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$   
 $\int_{-\infty}^{\infty} \delta(x) dx = 1$   
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$   
 $\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$

(5)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$   
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$   
 $\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$

(6)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$

(7)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$   
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$   
 $\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$

(8)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$   
 $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$





Example 1

Consider a function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^2 + 2x + 1$ . We want to show that  $f$  is continuous at  $x = 0$ . Let  $\epsilon > 0$  be given. We need to find  $\delta > 0$  such that if  $|x - 0| < \delta$ , then  $|f(x) - f(0)| < \epsilon$ . Note that  $f(0) = 1$ . So we need  $|x^2 + 2x + 1 - 1| < \epsilon$ , which simplifies to  $|x^2 + 2x| < \epsilon$ . We can choose  $\delta = \min\{1, \epsilon/3\}$ . Then if  $|x| < \delta$ , we have  $|x^2 + 2x| \leq |x|^2 + 2|x| < \delta^2 + 2\delta < \delta + 2\delta = 3\delta \leq \epsilon$ .

Consider a function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^2 + 2x + 1$ . We want to show that  $f$  is continuous at  $x = 0$ . Let  $\epsilon > 0$  be given. We need to find  $\delta > 0$  such that if  $|x - 0| < \delta$ , then  $|f(x) - f(0)| < \epsilon$ . Note that  $f(0) = 1$ . So we need  $|x^2 + 2x + 1 - 1| < \epsilon$ , which simplifies to  $|x^2 + 2x| < \epsilon$ . We can choose  $\delta = \min\{1, \epsilon/3\}$ . Then if  $|x| < \delta$ , we have  $|x^2 + 2x| \leq |x|^2 + 2|x| < \delta^2 + 2\delta < \delta + 2\delta = 3\delta \leq \epsilon$ .

Example 2



10.  $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = -\frac{1}{x} + C$

10.  $\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-3+1}}{-3+1} = \frac{x^{-2}}{-2} = -\frac{1}{2x^2} + C$

10.  $\int \frac{1}{x^4} dx = \int x^{-4} dx = \frac{x^{-4+1}}{-4+1} = \frac{x^{-3}}{-3} = -\frac{1}{3x^3} + C$

10.  $\int \frac{1}{x^5} dx = \int x^{-5} dx = \frac{x^{-5+1}}{-5+1} = \frac{x^{-4}}{-4} = -\frac{1}{4x^4} + C$

10.  $\int \frac{1}{x^6} dx = \int x^{-6} dx = \frac{x^{-6+1}}{-6+1} = \frac{x^{-5}}{-5} = -\frac{1}{5x^5} + C$

- (1)  $\int \frac{1}{x^7} dx = \int x^{-7} dx = \frac{x^{-7+1}}{-7+1} = \frac{x^{-6}}{-6} = -\frac{1}{6x^6} + C$
- (2)  $\int \frac{1}{x^8} dx = \int x^{-8} dx = \frac{x^{-8+1}}{-8+1} = \frac{x^{-7}}{-7} = -\frac{1}{7x^7} + C$
- (3)  $\int \frac{1}{x^9} dx = \int x^{-9} dx = \frac{x^{-9+1}}{-9+1} = \frac{x^{-8}}{-8} = -\frac{1}{8x^8} + C$
- (4)  $\int \frac{1}{x^{10}} dx = \int x^{-10} dx = \frac{x^{-10+1}}{-10+1} = \frac{x^{-9}}{-9} = -\frac{1}{9x^9} + C$
- (5)  $\int \frac{1}{x^{11}} dx = \int x^{-11} dx = \frac{x^{-11+1}}{-11+1} = \frac{x^{-10}}{-10} = -\frac{1}{10x^{10}} + C$
- (6)  $\int \frac{1}{x^{12}} dx = \int x^{-12} dx = \frac{x^{-12+1}}{-12+1} = \frac{x^{-11}}{-11} = -\frac{1}{11x^{11}} + C$
- (7)  $\int \frac{1}{x^{13}} dx = \int x^{-13} dx = \frac{x^{-13+1}}{-13+1} = \frac{x^{-12}}{-12} = -\frac{1}{12x^{12}} + C$
- (8)  $\int \frac{1}{x^{14}} dx = \int x^{-14} dx = \frac{x^{-14+1}}{-14+1} = \frac{x^{-13}}{-13} = -\frac{1}{13x^{13}} + C$
- (9)  $\int \frac{1}{x^{15}} dx = \int x^{-15} dx = \frac{x^{-15+1}}{-15+1} = \frac{x^{-14}}{-14} = -\frac{1}{14x^{14}} + C$
- (10)  $\int \frac{1}{x^{16}} dx = \int x^{-16} dx = \frac{x^{-16+1}}{-16+1} = \frac{x^{-15}}{-15} = -\frac{1}{15x^{15}} + C$

110

Handwritten text for problem 110, consisting of several lines of mathematical reasoning or calculations.

111

Handwritten text for problem 111, consisting of several lines of mathematical reasoning or calculations.

6  $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$  ...

112

Handwritten text for problem 112, consisting of several lines of mathematical reasoning or calculations.

Handwritten text for problem 112, consisting of several lines of mathematical reasoning or calculations.

Handwritten text for problem 112, consisting of several lines of mathematical reasoning or calculations.

113

Handwritten text for problem 113, consisting of several lines of mathematical reasoning or calculations.

(1) Handwritten text for problem 113, consisting of several lines of mathematical reasoning or calculations.

(2) Handwritten text for problem 113, consisting of several lines of mathematical reasoning or calculations.

(3) Handwritten text for problem 113, consisting of several lines of mathematical reasoning or calculations.

(4) Handwritten text for problem 113, consisting of several lines of mathematical reasoning or calculations.

(5) Handwritten text for problem 113, consisting of several lines of mathematical reasoning or calculations.

(6) Handwritten text for problem 113, consisting of several lines of mathematical reasoning or calculations.



exmp 11

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exmp 11

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exmp 11

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exmp 11

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exmp 120

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exp 12  
The first part of the experiment is to determine the effect of temperature on the rate of reaction. The reaction is the reaction between hydrogen peroxide and potassium iodide. The rate of reaction is measured by the volume of oxygen gas produced over a fixed time interval. The results show that the rate of reaction increases with increasing temperature.

exp 12  
The second part of the experiment is to determine the effect of concentration on the rate of reaction. The reaction is the reaction between hydrogen peroxide and potassium iodide. The rate of reaction is measured by the volume of oxygen gas produced over a fixed time interval. The results show that the rate of reaction increases with increasing concentration of hydrogen peroxide.

exp 12  
The third part of the experiment is to determine the effect of catalyst on the rate of reaction. The reaction is the reaction between hydrogen peroxide and potassium iodide. The rate of reaction is measured by the volume of oxygen gas produced over a fixed time interval. The results show that the rate of reaction increases with the addition of a catalyst.



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Ex 1, 2

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exmp 1, •

... (2) ... (11) ... (12) ... exmp 1, 2. ...

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exmp 1, •

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exmp 1,

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exmp 1,

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exp 1, -

(10)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (1)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (2)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$

(11)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (2)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (3)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$

(12)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (4)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (5)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$

$\int_{-\infty}^{\infty} \delta(x) dx = 1$  (6)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (7)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (8)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (9)

1.  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (10)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (11)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (12)

1.0  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (13)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (14)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (15)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (16)

$\int_{-\infty}^{\infty} \delta(x) dx = 1$  (17)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (18)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (19)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (20)

1.1  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (21)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (22)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (23)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (24)

1.2  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (25)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (26)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (27)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (28)

1.  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (29)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (30)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (31)

(1)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (32)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (33)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (34)

(2)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (35)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (36)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (37)





- (•)  $\int_{-\infty}^{\infty} \delta(x) dx = 1$  (normalization)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x dx = 0$  (odd function)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x^2 dx = 0$  (odd function)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x^3 dx = 0$  (odd function)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x^4 dx = 0$  (odd function)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x^5 dx = 0$  (odd function)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x^6 dx = 0$  (odd function)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x^7 dx = 0$  (odd function)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x^8 dx = 0$  (odd function)
- ( )  $\int_{-\infty}^{\infty} \delta(x) x^9 dx = 0$  (odd function)
- (10)  $\int_{-\infty}^{\infty} \delta(x) x^{10} dx = 0$  (odd function)
- (11)  $\int_{-\infty}^{\infty} \delta(x) x^{11} dx = 0$  (odd function)
- (12)  $\int_{-\infty}^{\infty} \delta(x) x^{12} dx = 0$  (odd function)
- (1•)  $\int_{-\infty}^{\infty} \delta(x) x^{13} dx = 0$  (odd function)
- (1, )  $\int_{-\infty}^{\infty} \delta(x) x^{14} dx = 0$  (odd function)
- (1 )  $\int_{-\infty}^{\infty} \delta(x) x^{15} dx = 0$  (odd function)
- (1 )  $\int_{-\infty}^{\infty} \delta(x) x^{16} dx = 0$  (odd function)
- (1 )  $\int_{-\infty}^{\infty} \delta(x) x^{17} dx = 0$  (odd function)
- (1 )  $\int_{-\infty}^{\infty} \delta(x) x^{18} dx = 0$  (odd function)
- (1 )  $\int_{-\infty}^{\infty} \delta(x) x^{19} dx = 0$  (odd function)

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exmp 1, ...

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1.0  $\int_{-1}^1 (x^2 + 1) dx = \int_{-1}^1 x^2 dx + \int_{-1}^1 1 dx = \left[ \frac{x^3}{3} + x \right]_{-1}^1 = \left( \frac{1}{3} + 1 \right) - \left( -\frac{1}{3} - 1 \right) = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$

- 1.1
- (1)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (2)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (3)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (4)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (5)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (6)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (7)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$

1.2  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$

1.  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$

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- (1)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (2)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (3)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$
  - (4)  $\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$

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(•)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^+) + f(a^-))$ .

(.)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
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(.)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
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If  $f$  has an essential discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx$  is not defined.

(.)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^+) + f(a^-))$ .

201  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
(1)  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .

202  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^+) + f(a^-))$ .

20•  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^+) + f(a^-))$ .  
If  $f$  has a removable discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$ .  
If  $f$  has an essential discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx$  is not defined.

20,  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^+) + f(a^-))$ .

$\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$  if  $f$  is continuous at  $a$ .  
If  $f$  has a jump discontinuity at  $a$ , then  $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \frac{1}{2}(f(a^+) + f(a^-))$ .

exmp 20

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exmp 20

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exmp 20

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exmp 20

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- (.) ...
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exmp 20

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- (1) ...
- (2) ...

- (\*)  $\text{supp}(f) \cap \text{supp}(g) = \emptyset$  if and only if  $\text{supp}(f+g) = \text{supp}(f) \cup \text{supp}(g)$ .
- (\*)  $\text{supp}(f) \cap \text{supp}(g) \neq \emptyset$  if and only if  $\text{supp}(f+g) \subsetneq \text{supp}(f) \cup \text{supp}(g)$ .
- (\*)  $\text{supp}(f) \cap \text{supp}(g) = \emptyset$  if and only if  $\text{supp}(fg) = \text{supp}(f) \cap \text{supp}(g)$ .
- (\*)  $\text{supp}(f) \cap \text{supp}(g) \neq \emptyset$  if and only if  $\text{supp}(fg) \subsetneq \text{supp}(f) \cap \text{supp}(g)$ .
- (\*)  $\text{supp}(f) \cap \text{supp}(g) = \emptyset$  if and only if  $\text{supp}(f+g) = \text{supp}(f) \cup \text{supp}(g)$  and  $\text{supp}(fg) = \text{supp}(f) \cap \text{supp}(g)$ .
- (\*)  $\text{supp}(f) \cap \text{supp}(g) \neq \emptyset$  if and only if  $\text{supp}(f+g) \subsetneq \text{supp}(f) \cup \text{supp}(g)$  and  $\text{supp}(fg) \subsetneq \text{supp}(f) \cap \text{supp}(g)$ .
- (\*)  $\text{supp}(f) \cap \text{supp}(g) = \emptyset$  if and only if  $\text{supp}(f+g) = \text{supp}(f) \cup \text{supp}(g)$  and  $\text{supp}(fg) = \text{supp}(f) \cap \text{supp}(g)$ .
- (\*)  $\text{supp}(f) \cap \text{supp}(g) \neq \emptyset$  if and only if  $\text{supp}(f+g) \subsetneq \text{supp}(f) \cup \text{supp}(g)$  and  $\text{supp}(fg) \subsetneq \text{supp}(f) \cap \text{supp}(g)$ .
- (10)  $\text{supp}(f) \cap \text{supp}(g) = \emptyset$  if and only if  $\text{supp}(f+g) = \text{supp}(f) \cup \text{supp}(g)$  and  $\text{supp}(fg) = \text{supp}(f) \cap \text{supp}(g)$ .

$\text{supp}(f) \cap \text{supp}(g) = \emptyset$  if and only if  $\text{supp}(f+g) = \text{supp}(f) \cup \text{supp}(g)$  and  $\text{supp}(fg) = \text{supp}(f) \cap \text{supp}(g)$ .



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- (1)
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  - (3)

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The first part of the exam is a multiple choice section. The first question is about the definition of a function. The correct answer is (C).

22

The second question is about the domain of a function. The correct answer is (B).

22

The third question is about the range of a function. The correct answer is (D).

22

The fourth question is about the composition of functions. The correct answer is (A).

(1) The first question is about the definition of a function. The correct answer is (C).

(2) The second question is about the domain of a function. The correct answer is (B).

(3) The third question is about the range of a function. The correct answer is (D).

(4) The fourth question is about the composition of functions. The correct answer is (A).

(5) The fifth question is about the definition of a function. The correct answer is (C).

(6) The sixth question is about the domain of a function. The correct answer is (B).

(7) The seventh question is about the range of a function. The correct answer is (D).

(8) The eighth question is about the composition of functions. The correct answer is (A).

(9) The ninth question is about the definition of a function. The correct answer is (C).

(10) The tenth question is about the domain of a function. The correct answer is (B).

(11) The eleventh question is about the range of a function. The correct answer is (D).

(12) The twelfth question is about the composition of functions. The correct answer is (A).

22

The final question is about the composition of functions. The correct answer is (A).



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Ex 2, (2)  $\frac{1}{2} \int_0^1 (x^2 + 2x + 1) dx = \frac{1}{2} \left[ \frac{x^3}{3} + x^2 + x \right]_0^1 = \frac{1}{2} \left( \frac{1}{3} + 1 + 1 \right) = \frac{1}{2} \left( \frac{7}{3} \right) = \frac{7}{6}$

Ex 2, (1)  $\int_0^1 (x^2 + 2x + 1) dx = \left[ \frac{x^3}{3} + x^2 + x \right]_0^1 = \frac{1}{3} + 1 + 1 = \frac{7}{3}$

(1)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(2)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(3)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(4)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(5)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(6)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(7)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(8)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(9)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

(10)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

Ex 2 0 (1)  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

Ex 2 1  $\int_0^1 (x^2 + 2x + 1) dx = \frac{7}{3}$

Ex 2.2. (a) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2 + 1$ . Show that  $f$  is not a linear transformation.

(b) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 2x$ . Show that  $f$  is a linear transformation.

Ex 2.3. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2$ . Show that  $f$  is not a linear transformation.

Ex 2.4. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x$ . Show that  $f$  is a linear transformation.

(1)  $f(x+y) = f(x) + f(y)$

(2)  $f(ax) = a f(x)$

(3)  $f(0) = 0$

### 3

Ex 2.5. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2 + 2x$ . Show that  $f$  is not a linear transformation.

Ex 2.6. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 3x$ . Show that  $f$  is a linear transformation.

Ex 2.7. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2 + x$ . Show that  $f$  is not a linear transformation.

$$f(x+y) = (x+y)^2 + (x+y) = x^2 + 2xy + y^2 + x + y$$
$$= x^2 + x + y^2 + y + 2xy = f(x) + f(y) + 2xy$$

Ex 2.8. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2$ . Show that  $f$  is not a linear transformation.

Ex 2.9. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x$ . Show that  $f$  is a linear transformation.

(1)  $f(x+y) = f(x) + f(y)$

(2)  $f(ax) = a f(x)$

- (\*) ...
- (\*) ...

ex 2 0

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ex 2 1

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- (1) ...
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- (10) ...

(11)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$  because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively. The only non-zero value of  $\delta(x)$  is at  $x=0$  and the only non-zero value of  $\delta(x-a)$  is at  $x=a$ .

(12) The integral  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx$  is zero because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively. The only non-zero value of  $\delta(x)$  is at  $x=0$  and the only non-zero value of  $\delta(x-a)$  is at  $x=a$ .

- (i)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$
- (ii)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$
- (iii)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$

exmp 2.2 The integral  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx$  is zero because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively. The only non-zero value of  $\delta(x)$  is at  $x=0$  and the only non-zero value of  $\delta(x-a)$  is at  $x=a$ .

(1)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$  because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively.

(2)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$  because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively. (1) is correct.

(\*)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$  because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively. (1) and (2) are correct.

(\*)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$  because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively. (1), (2) and (\*) are correct.

(\*)  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx = 0$  because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively. (\*) is correct.

exmp 2.3 The integral  $\int_{-\infty}^{\infty} \delta(x) \delta(x-a) dx$  is zero because  $\delta(x)$  and  $\delta(x-a)$  are both zero at  $x=0$  and  $x=a$  respectively. The only non-zero value of  $\delta(x)$  is at  $x=0$  and the only non-zero value of  $\delta(x-a)$  is at  $x=a$ .





Ex 2 •

1. Theorem: Let  $f: X \rightarrow Y$  be a function. Then  $f$  is injective if and only if  $f(A \cap B) = f(A) \cap f(B)$  for all subsets  $A, B$  of  $X$ .

- (1)  $f(A \cap B) \subseteq f(A) \cap f(B)$  for all subsets  $A, B$  of  $X$ .
- (2)  $f(A) \cap f(B) \subseteq f(A \cap B)$  for all subsets  $A, B$  of  $X$  if and only if  $f$  is injective.
- (3)  $f(A \cup B) \supseteq f(A) \cup f(B)$  for all subsets  $A, B$  of  $X$ .
- (4)  $f(A) \cup f(B) \subseteq f(A \cup B)$  for all subsets  $A, B$  of  $X$ .
- (5)  $f(A \cap B) = f(A) \cap f(B)$  for all subsets  $A, B$  of  $X$  if and only if  $f$  is injective.

Ex 2 •

Let  $f: X \rightarrow Y$  be a function. Then  $f$  is surjective if and only if  $f(X) = Y$ .

Let  $f: X \rightarrow Y$  be a function. Then  $f$  is bijective if and only if  $f$  is both injective and surjective.

- (1)  $f$  is surjective if and only if  $f(X) = Y$ .
- (2)  $f$  is bijective if and only if  $f$  is both injective and surjective.
- (3)  $f$  is injective if and only if  $f(A \cap B) = f(A) \cap f(B)$  for all subsets  $A, B$  of  $X$ .
- (4)  $f$  is surjective if and only if  $f(X) = Y$ .
- (5)  $f$  is bijective if and only if  $f$  is both injective and surjective.

exmp 2

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(1) ... ..

(2) ... ..

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$$y = 10 \dots \dots \dots$$

$$\dots \dots \dots$$

exmp 2

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exmp 2

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exmp 2,

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exmp 2

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(1) ... ..

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exmp 2-

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(1) ...  
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(2) ...  
 12- ...

$$\sqrt{11} \cdot 2 \cdot \sqrt{11} \cdot \sqrt{11}$$

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$$\sqrt{11} \cdot 3 \cdot \sqrt{11} \cdot \sqrt{11} \cdot \sqrt{11} \cdot \sqrt{11} \cdot \sqrt{11}$$

... 2 1 ...  
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Ex 1.0, ...

Ex 2.0, ...

... ( ) ...

Ex 3.0, ...

Ex 4.0, ...

2

Ex 5.0, ...

•0

Handwritten musical notation for the first system, including a treble clef, a key signature of one flat, and a common time signature. The notation consists of a single staff with various notes and rests.

12

1

•10

Handwritten musical notation for the second system, continuing the piece with a treble clef and common time signature.

Handwritten musical notation for the third system, continuing the piece with a treble clef and common time signature.

•11

Handwritten musical notation for the fourth system, continuing the piece with a treble clef and common time signature.

•12

Handwritten musical notation for the fifth system, including a treble clef, a key signature of one flat, and a common time signature. The notation includes a measure with the number 10 written above it.

•1

Handwritten musical notation for the sixth system, continuing the piece with a treble clef and common time signature.

•1,

Handwritten musical notation for the seventh system, including a treble clef, a key signature of one flat, and a common time signature. The notation includes a measure with the number 10 written above it.

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10

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(1)

(2)

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( ) 10%

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(1) (2) ...

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(1) ...

(2) ...

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exmp •2•

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exmp •2,

exmp •2

exer •2

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exer •2

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exer •2

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13

exer •2

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(1) ... ..

(2) ... ..

(3) ... ..

exer ••0

... ..

1.  $\frac{1}{x^2} = x^{-2}$ ,  $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$   
 $\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

2.  $\frac{1}{x^3} = x^{-3}$ ,  $\frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$   
 $\frac{d}{dx} \frac{1}{x^3} = -\frac{3}{x^4}$

3.  $\frac{1}{x^4} = x^{-4}$ ,  $\frac{d}{dx} x^{-4} = -4x^{-5} = -\frac{4}{x^5}$   
 $\frac{d}{dx} \frac{1}{x^4} = -\frac{4}{x^5}$

$$y = \frac{1}{x^2} = 14 \quad x^2 \quad x^3 \quad x^4$$

4.  $\frac{1}{x^5} = x^{-5}$ ,  $\frac{d}{dx} x^{-5} = -5x^{-6} = -\frac{5}{x^6}$   
 $\frac{d}{dx} \frac{1}{x^5} = -\frac{5}{x^6}$

(1)  $\frac{d}{dx} \frac{1}{x^6} = \frac{d}{dx} x^{-6} = -6x^{-7} = -\frac{6}{x^7}$   
 $\frac{d}{dx} \frac{1}{x^6} = -\frac{6}{x^7}$   
 $\frac{d}{dx} \frac{1}{x^7} = \frac{d}{dx} x^{-7} = -7x^{-8} = -\frac{7}{x^8}$   
 $\frac{d}{dx} \frac{1}{x^7} = -\frac{7}{x^8}$   
 $\frac{d}{dx} \frac{1}{x^8} = \frac{d}{dx} x^{-8} = -8x^{-9} = -\frac{8}{x^9}$   
 $\frac{d}{dx} \frac{1}{x^8} = -\frac{8}{x^9}$   
 $\frac{d}{dx} \frac{1}{x^9} = \frac{d}{dx} x^{-9} = -9x^{-10} = -\frac{9}{x^{10}}$   
 $\frac{d}{dx} \frac{1}{x^9} = -\frac{9}{x^{10}}$

$\frac{d}{dx} \frac{1}{x^{10}} = \frac{d}{dx} x^{-10} = -10x^{-11} = -\frac{10}{x^{11}}$   
 $\frac{d}{dx} \frac{1}{x^{10}} = -\frac{10}{x^{11}}$   
 $\frac{d}{dx} \frac{1}{x^{11}} = \frac{d}{dx} x^{-11} = -11x^{-12} = -\frac{11}{x^{12}}$   
 $\frac{d}{dx} \frac{1}{x^{11}} = -\frac{11}{x^{12}}$   
 $\frac{d}{dx} \frac{1}{x^{12}} = \frac{d}{dx} x^{-12} = -12x^{-13} = -\frac{12}{x^{13}}$   
 $\frac{d}{dx} \frac{1}{x^{12}} = -\frac{12}{x^{13}}$

$\frac{d}{dx} \frac{1}{x^{13}} = \frac{d}{dx} x^{-13} = -13x^{-14} = -\frac{13}{x^{14}}$   
 $\frac{d}{dx} \frac{1}{x^{13}} = -\frac{13}{x^{14}}$

(2)  $\frac{d}{dx} \frac{1}{x^{14}} = \frac{d}{dx} x^{-14} = -14x^{-15} = -\frac{14}{x^{15}}$   
 $\frac{d}{dx} \frac{1}{x^{14}} = -\frac{14}{x^{15}}$

(•)  $\frac{1}{2} \times 100 = 50\%$  (1)  $\frac{1}{2} \times 100 = 50\%$

(•)  $\frac{1}{2} \times 100 = 50\%$

15

••

(1)  $0\%$   $0\%$

(2)

(•)

(•)

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